Gravitational focusing of cosmic neutrinos by the solar interior

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The Sun is transparent for cosmic neutrinos with an energy less than 80 GeV. The interior part of the Sun forms for this radiation a high-quality gravitational lens with the focal distance $F=24$ AU (astronomical units)—the distance between the Uranus and Neptune orbit radii. The factors distorting the focusing are estimated. Among them are the spherical aberration, astigmatism caused by the rotation of the Sun, fluctuations of density connected with convection, and sunspots, etc. The lens has the unprecedented angular resolution about 1.4×10^{-11} rad. In focus the increase in intensity for the neutrinos flux from the center of Sirius is about a million times and even larger for more distant objects. This is only two orders of magnitude less than the flux of solar neutrinos on the Earth. Information about the solar interior can also be obtained by such observations. The resulting increase in intensity of the neutrino flux from the Sirius seems to open great possibilities in their detection even at the present virtual level of registration technique.

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I. INTRODUCTION

The detection of cosmic neutrinos from pointlike sources could give important information unobtainable by other methods. The brightest sources registered up to now are the Sun and the Supernova in the Large Magellanic Cloud SN 1987A, which were thoroughly discussed by Bahcall [1]. Other sources of neutrinos are enormously weaker (10^{10} times and more) and at first sight seem completely unobservable. That is why they were not considered in [1]. However there is a possibility to compensate almost completely this enormous factor using an extremely small solid angle from which this radiation comes, i.e., to use the focusing principle. The focusing of neutrinos appears again impossible because of their weak interaction with matter. Therefore we need as a focusing device the biggest available object such as the Sun.

In this paper we consider the possibility to increase the flux of neutrinos from the star-like objects by many orders of magnitude using the high quality gravitational focusing properties of solar *interior*. Originally the idea of gravitational focusing was proposed by Einstein and Fogt in 1936 [2], but they considered only *exterior* focusing of the rays of light by gravitational field of the Sun. In this case the deflection angle $\delta\varphi_{ext}(\rho)$ is inversely proportional to the impact parameter ρ:

$$\delta\varphi_{ext}(\rho)=\frac{4GM_{\odot}}{c^2\rho}, \tag{1}$$

where $M_{\odot}$ is the mass of the Sun, G the gravitational constant, and c the speed of light. Such a lens is of a low quality because it smoothed the focal distance F for the parallel beam of rays, starting from F_{min} corresponding to the rays touching the disk of the star and up to infinity for larger ρ. For the Sun,

$$F_{min}=\frac{R_{\odot}}{\delta\varphi_{\odot ext}}=548.4 \ \ \mathrm{AU}, \tag{2}$$

where $\delta\varphi_{\odot ext}=1.75''$ is one of the main experimentally observed values confirming Einstein's general theory of relativity. This distance F_{min} is far beyond the solar system and unattainable at present by space missions.

The good lens properties of the interior gravitational field of the massive cylindrically and spherically symmetric object (e.g. the star) with respect to the gravitational radiation for the greatly general assumptions about a radial dependence of the mass density was first pointed out in [3,4]. In particular, the same gravitational effect has been discussed in [3] in order to disprove the results of Weber's experiment [5] on detection of gravitational radiation focused by the galactic core. The Sun as the focusing object and neutrinos as the focused radiation were not considered in [3,4].

II. THE DEFLECTION ANGLE

Let us consider the small-angle deflection of cosmic neutrino rays by an internal gravitational field of a spherically symmetrical star. Then, the formula for the internal deflection angle $\delta\varphi_{int}(\rho)$ remains the same as Eq. (1), but the mass $M_{\odot}$ should be replaced by a total mass $M(\rho)$ contained within an infinite cylinder of radius ρ with the axis passing through the center of masses parallel to the incident beam:

$$\delta\varphi_{int}(\rho)=\frac{4GM(\rho)}{c^2\rho}, \tag{3}$$

where

$$M(\rho)=M-4\pi\int_{\rho}^{\infty}\mu(r)r^2\sqrt{1-\frac{\rho^2}{r^2}}dr, \tag{4}$$

and $\mu(r)$ is the mass density. Speaking otherwise, all masses within the cylinder can be added and put at the origin; all masses outside the cylinder compensate each other and do not affect the deflection [3,4]. So, knowing the spherically symmetrical distribution of masses within the star and calculating the total mass $M(\rho)$ within the cylinder of radius ρ it is easy to get $\delta\varphi_{int}(\rho)$. The good focusing properties exist only if $\delta\varphi_{int}(\rho)$ is proportional to ρ which is correct only for small values of ρ. Therefore we expand $\delta\varphi_{int}(\rho)$ in odd powers of ρ (the absence of even powers is evident from the symmetry considerations) and keep only the two first terms:

$$\delta\varphi_{int}(\rho)=\alpha_1\left(\frac{\rho}{R}\right)+\alpha_3\left(\frac{\rho}{R}\right)^3+O\left(\left(\frac{\rho}{R}\right)^5\right). \quad (5)$$

We introduce here the value R—the visible radius of the star to make the coefficients α_1,α_3 dimensionless. For determination of the coefficients α_1,α_3 one can use the Taylor formula for $M(\rho)$ in a vicinity at the point $\rho=0$

$$M(\rho)=M(0)+M'(0)\rho+\frac{M''(0)}{2!}\rho^2+\frac{M'''(0)}{3!}\rho^3+\frac{M^{(4)}(0)}{4!}\rho^4+o(\rho^5). \quad (6)$$

It is obvious that $M(0)=0$. The coefficients $M^{(i)}(0)$ are equal to the values of derivatives of the ith order of function $M(\rho)$ in the point $\rho=0$, the derivatives being obtained from Eq. (4). After some transformations we arrive at the following expressions:

$$M'(0)=M'''(0)=0, \quad M''(0)=4\pi\int_0^\infty \mu(r)dr,$$

$$M^{(4)}(0)=12\pi\int_0^\infty \frac{d\mu}{dr}\frac{dr}{r}. \quad (7)$$

Calculating $M^{(4)}(0)$ we assumed that $\mu'(0)=0$. The derivative $(d\mu/dr)$ vanishes at the lower integration limit which makes the integral finite. Inserting Eqs. (6) and (7) into Eq. (3) we get

$$\alpha_1=\frac{8\pi GR}{c^2}\int_0^\infty \mu(r)dr, \quad \alpha_3=\frac{2\pi GR^3}{c^2}\int_0^\infty \frac{d\mu}{dr}\frac{dr}{r}. \quad (8)$$

Calculating α_1 we make the simplest assumption that $\mu(r)$ is constant in each of transverse sections of the cylinder. Calculating α_3 we take into account the change of density within each transverse section.[1] The focal distance of the internal gravitational lens is

$$F=\frac{R}{\alpha_1}. \quad (9)$$

The cubic term in Eq. (5) spoils the ideal focusing and restricts the effective radius of the lens producing the main part of the spherical aberration.

The gravitational focusing by a spherically symmetrical star has been considered also in [6]. However the author admitted completely unrealistic density distributions ($\mu(r)=\mu_0$, i.e., a constant one and $\mu(r)=\mu_0(1-r^2/R^2)$), neglecting the strong concentration of density in the star's center (see Fig. 1). Correspondingly, in the case of the Sun, these distributions provide the focal distances calculated according to the formula (9), which the values 366 AU and 219 AU. These distances are too large and they are placed outside the solar system. The main difference of these distributions from the real one given by the standard solar model (SSM) [1] is the weak concentration of masses to the center. As seen from Fig. 1, the gravity for the SSM reaches its maximum value at $r=0.17R_\odot$ and becomes 8.5 times as large as the surface gravity. Using the SSM we have calculated the deflection angle $\delta\varphi_{int}(\rho)$ given in Fig. 2 and coefficients α_1,α_3:

$$\alpha_1=1.9389\times10^{-4}, \quad \alpha_3=-4.284\times10^{-3}. \quad (10)$$

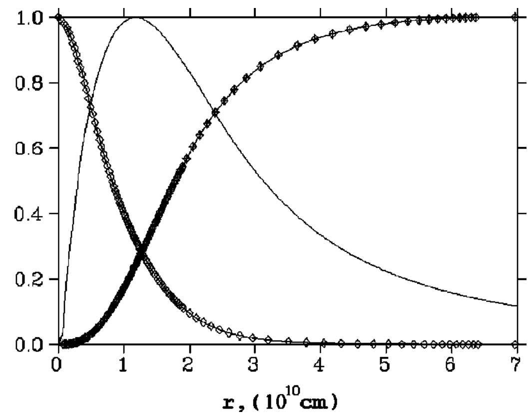

FIG. 1. *Empty diamonds line*: the standard solar model (SSM) relative density distribution $\mu(r)/\mu(0)$ [$\mu(0)=148$ g/cm^3]. The density decreases twice from the center to the $r=0.12R_\odot$ ($R_\odot=6.96\times10^{10}$ cm) which shows the degree of the mass concentration. *Crossed diamonds line*: SSM relative total mass within the sphere of radius r: $M(r)/M_\odot$ ($M_\odot=1.99\times10^{33}$ g). *Solid line*: the relative gravity $g(r)/g_{max}$ ($r_{max}=0.17R_\odot$; $g_{max}=2.33\times10^5$ cm/c^2) which is 8.5 times more than $g(R_\odot)$.

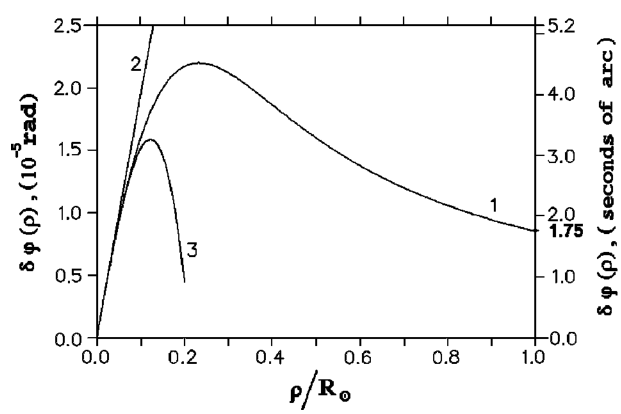

FIG. 2. The deflection angle as a function of the relative impact parameter $\rho/R_\odot$. (1) Calculations along the formula (3) for the SSM. (2) The linear approximation. (3) The cubic approximation. One can see that the deviations from the linear approximation (ideal focusing) starts already at $r=R_\odot/20$. Different scales for $\delta\varphi(\rho)$ are given at the left and right sides of the figure.

[1]If $\mu(r)$ is a constant for $r<R$ and $\mu=0$ for $r>R$ then the formula (8) for α_3 is still valid. In this case $\mu(r)$ undergoes a jump when $r=R$ and $(d\mu/dr)_{r=R}=\mu_0\delta(r-R)$. We still obtain the correct result.

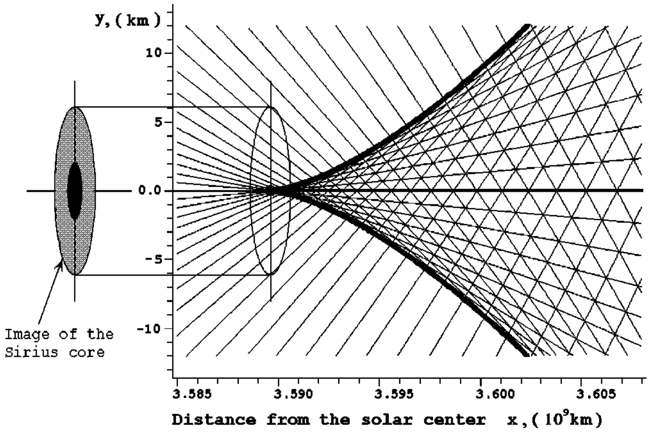

FIG. 3. The straight line rays in the vicinity of the focus calculated with account of the cubic term in Eq. (11). The intensity increase can be seen qualitatively as a density of lines condensed at one side of caustic. In the focal plane the singularity is proportional to $y^{-4/3}$. After the rotation around the x axis we will get on it a weaker (y^{-1}) singularity for $x>F$. The image of the Sirius core is also drawn as a circle of 6.1 km radius in the focal plane. The black area indicates the further contraction of the image when nonuniform distribution of neutrino intensity is taken into account. Note the drastic difference in scales along the x and y axes. This extends the image along the x axis up to millions of km s and only the problem of finding the image in the transverse plane remains.

In Fig. 2 the lines $\delta\varphi_1=\alpha_1\cdot(\rho/R_\odot)$ and $\delta\varphi_3=\alpha_1\cdot(\rho/R_\odot)+\alpha_3\cdot(\rho/R_\odot)^3$ are also shown. It is easy to see that the linear approximation is valid in a relatively small central region of the Sun—for ρ less than one twentieth of the $R_\odot$. Using Eqs. (9) and (10) we get for the focal distance

$$F=24\ \text{AU}.$$

This distance is then within the range of the solar system: between the radii of the orbits of Uranus (19 AU) and Neptune (30 AU), which is drastically closer than F_{min} due to the concentration of masses mentioned above.

The family of neutrino trajectories at large distances will be an axially-symmetrical family of straight lines

$$y(x)=\rho-x\left(\alpha_1\left(\frac{\rho}{R_\odot}\right)+\alpha_3\left(\frac{\rho}{R_\odot}\right)^3\right), \tag{11}$$

where x is the distance from the solar center, y—the distance from the axis in the cylindrical coordinates in the vicinity of the focus (see Fig. 3). This family of straight lines transforms into a conical one if α_3 is negligible. Then we have the ideal focusing. In general when $\alpha_3\neq 0$ there exists the spherical aberration and the trajectories form a caustic envelope, which can be obtained by the rotation of the semicubic parabola

$$y(x)=\frac{2}{3}\sqrt{\frac{(\alpha_1 x-R_\odot)^3}{3|\alpha_3|x}} \tag{12}$$

around the x axis. The intensity of the neutrino flux becomes infinite on the surface of caustic as well as in the focus and along the x axis. The caustic, the focus and the straight-line trajectories are presented in Fig. 3. The increase in intensity on the focal plane perpendicular to the x axis is given by the formula

$$I(y)=\frac{1}{3}\left(\frac{\alpha_1}{|\alpha_3|}\right)^{2/3}\frac{R_\odot^{4/3}}{y^{4/3}}=\frac{2.61\times10^{10}}{y^{4/3}(m)}. \tag{13}$$

So it has the $y^{-4/3}$ singularity of intensity on the focal plane in the case of the pointlike source. This spherical aberration singularity should be smoothed further due to irregularities of solar interior and due to the finite angular size of the neutrino source.

Let us consider the Sirius as a neutrino stable source, the brightest one among nearest stars. First, for simplicity, we suppose that the brightness of the neutrino source does not depend upon the radius. The size of this power producing core is assumed to be $0.2R_\odot$ [7]. Thus, for the radius of the image on the focal plane we obtain the value of 6.1 km. Now we have to convolute both intensity distributions. Then we shall have the maximal intensity in the focus increased at least 10^6 times relative to the direct flux. For the possible starlike sources with smaller angular size their image will be smaller and therefore due to the $y^{-4/3}$ singularity the focusing concentration of intensity will be higher which partly compensate the geometrical decrease in brightness. Secondly, let us take into consideration the fact that the brightness of the neutrino producing core is nonuniform due to the very steep energy production—temperature dependence (as T^{20}). Therefore the effective radius of the core can be at least three times less than our initial assumption what increases the focusing concentration of the neutrino flux at least 5 times and may be more. In this estimation the model of the Sirius interior considered in [7] was used.

The spherical aberration is the most important factor limiting the quality of the gravitational solar lens. Another significant factor distorting the focusing is the fluctuations of density connected with the convection and sunspots. Assuming that the size of a convection cells is about 1000 km, the density variation within the cells is about 10%, the depth of convective zone is $0.26R_\odot$ and taking into account their partial regularity (relatively uniform distribution of granules on the solar surface) we get finally the total deflection of the neutrino rays on the focal plane of no more than 50 m. The influence of the sunspots can be easily estimated and it gives the deflection of a few meters.

The third distorting factor is the astigmatism of the lens connected with the rotation of the Sun and its nonsphericity. Assuming that the Sun rotates as a rigid body and taking into consideration that Sun's oblateness $\leqslant 2\times10^{-5}$ we have here the additional deflection of about 1 m in the focal plane.

We see that the irregular and therefore an unavoidable spreading of the image of the point-like source in the focal plane caused by convection is about 50 m. This is much less than the image of the Sirius core and we can therefore resolve the decrease in luminosity of the core. Besides we can state that our giant telescope has an unprecedented angular resolution of 1.4×10^{-11} rad when compared with any existing device.

In connection with the solar neutrino problem many authors have treated nonstandard solar models in which the core has a rapid rotation (see references in [1]). Assuming that the core rotates ten times as fast as the surface we obtain an additional deflection of the ray about 100 m on the focal plane, which could be observable and would give important information about this assumption. The weak influence of rotation is connected with its symmetry. The deviation from spherical symmetry in this case is of quadrupole nature and its action on the neutrino rays with small impact parameter vanishes in first approximation.

The last important factor connected with the possibility of the neutrino detection is the intensity of the neutrino emission, their energy spectrum and the corresponding sensitivity of various detectors.

Both the Sun and the Sirius generate energy by means of the *p*-*p* cycle and CNO cycle. However, the contribution of *p*-*p* cycle to the Sun's luminosity comprises more than 98% and that of CNO cycle is less than 2% [1]. But it is not the case with the Sirius where the contribution of *p*-*p* cycle is only about 25% and that of CNO cycle amounts to up to 75% [7]. The energy of neutrinos from *p*-*p* cycle is below 0.42 MeV. The energy of neutrinos from CNO cycle is below 1.74 MeV. Both types of neutrinos can be detected by ^{71}Ga detector [1] with the threshold of 0.23 MeV but those with higher energy are detected about ten times more efficiently.

All effects for the Sirius source can be summarized as follows: 3×10^{11} geometrical *decrease* in the brightness equal to the square of the relation of distances from the Sirius and from the Sun; 10^6 focusing *increase* factor for the uniform core; an *increase* factor of 5 connected with nonuniformity of the core; an *increase* factor of about 100 by which the Sirius is brighter than the Sun (including ultraviolet invisible part of Sirius radiation) and increased percent of neutrino radiation relative to the total one; an *increase* factor of about 10 due to different efficiency of ^{71}Ga detector for neutrinos from *p*-*p* and CNO cycles. On the whole our estimations are rather rough and we hope that with more exact calculations we might gain another order of magnitude.

Altogether the counting rate of neutrinos from the Sirius by such detector is about two orders of magnitude less than the counting rates of present Earth-bound detectors registering neutrinos from the Sun. (Note that the detector in the focus would detect solar neutrinos 580 times weaker than on the Earth and therefore interferes weakly with the Sirius flux.) We do not consider here the difficulties connected with the signal to noise ratio and the possibility to discriminate the cosmic rays false signals assuming that such an anticoincidence system (complicated enough) can be constructed. We also do not take into account all effects connected with possible neutrino oscillations.

III. CONCLUSION

To send a heavy neutrino detector to the outer part of the solar system actually to the Neptune orbit—this is a grandiose project that needs very serious argumentation. The detection of the Sirius neutrinos is probably an insufficient goal for such a project. However if stronger neutrino sources in the sky will be discovered, the situation could change. For instance, extremely bright (like a galaxy) and small (the size of the solar system) quasars can emit the energy mainly by neutrinos from the whole volume and only a small part by photons from the surface.

Eventually the solar neutrino telescope would give a unique chance to find and investigate such sources. Actually this is the unique way to increase the neutrino flux from pointlike sources and therefore sooner or later this project will be realized.

Unfortunately one needs years to bring the detector to the focus and therefore we cannot observe the initial stage of supernova explosions by such a telescope. In this connection an ideal object would be some novae, where the main part of the energy is emitted within several years to give time to reach the focus.

The quality of focusing can give direct information about the solar interior which is also a unique possibility.

All neutrino experiments are breathtaking because of the scales contrasts. The proposal discussed above gives another ultimate example of them.

The general idea of the neutrino focusing by the Sun was expressed by one of us (Yu.N.D.) and was presented in different lectures given in Sweden, Denmark, Japan, and Russia in 1994–1995 without calculations. It was based on the well known fact that good focusing preserves the luminosity of the object and is equivalent to the transfer of object (the Sirius) onto the lens place (the Sun) and the observer to the focus. All the calculations considered were performed by another author (A.M.P.) and they are presented here for the first time.

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